Delta Hedging with Support Vector Machines (SVM)

# Introduction

Delta hedging is a risk management strategy used to reduce the directional risk associated with options. It involves adjusting the position in the underlying asset to offset changes in the option's value due to price movements. By maintaining a delta-neutral portfolio, traders aim to protect against adverse price movements. This strategy requires continuous adjustments as the delta changes with market conditions.

# What is an Option?

An option is a financial contract that gives the holder the right, but not the obligation, to buy or sell an underlying asset at a specified price (strike price) before a certain date (expiration date). There are two types of options: calls (right to buy) and puts (right to sell). Options are used for hedging, speculation, and leverage. The price of an option is influenced by factors like the underlying asset price, volatility, time to expiration, and interest rates. Options provide flexibility but come with the risk of losing the premium paid if the market does not move as anticipated.

# Mathematical Concepts in Delta Hedging

## 1. Delta (Δ) Calculation:

Delta (Δ) is the first derivative of the option price (V) with respect to the price of the underlying asset (S):  
Δ = ∂V/∂S  
This measures the sensitivity of the option's price to small changes in the price of the underlying asset.

## 2. Black-Scholes Model:

The Black-Scholes model provides a closed-form solution for pricing European options and is used to calculate delta. For a call option, the delta is given by:  
Δ\_call = N(d1)  
where N(·) is the cumulative distribution function of the standard normal distribution and:  
d1 = [ln(S/K) + (r + 0.5σ²)T] / (σ√T)  
For a put option:  
Δ\_put = N(d1) - 1

## 3. Risk-Neutral Hedging:

Delta hedging aims to create a delta-neutral portfolio, where the combined delta of the option and the underlying asset position is zero. This is achieved by adjusting the holdings in the underlying asset to offset changes in the option's value.

## 4. Hedging Portfolio:

To hedge an option position, the trader holds -Δ shares of the underlying asset. The portfolio value (P) at any time (t) is:  
P\_t = V\_t - ΔS\_t  
where V\_t is the option value at time t, and S\_t is the price of the underlying asset at time t.

## 5. Rebalancing:

Since delta changes as the underlying asset price, time to expiration, volatility, and interest rates change, the hedge must be rebalanced periodically to maintain a delta-neutral position. The frequency of rebalancing can significantly affect the performance and cost of the hedging strategy.

## 6. Transaction Costs:

Rebalancing incurs transaction costs, which can be factored into the hedging strategy. The cost of adjusting the hedge position is proportional to the number of shares bought or sold.

# Mathematical Implementation in Code

Here’s a brief code snippet showing how delta might be calculated using the Black-Scholes model and used in delta hedging:

import numpy as np  
from scipy.stats import norm  
  
# Black-Scholes delta calculation for a call option  
def black\_scholes\_delta(S, K, T, r, sigma):  
 d1 = (np.log(S / K) + (r + 0.5 \* sigma\*\*2) \* T) / (sigma \* np.sqrt(T))  
 delta = norm.cdf(d1)  
 return delta  
  
# Example parameters  
S = 100 # Current price of the underlying asset  
K = 100 # Strike price of the option  
T = 1 # Time to expiration in years  
r = 0.05 # Risk-free interest rate  
sigma = 0.2 # Volatility of the underlying asset  
  
# Calculate delta  
delta = black\_scholes\_delta(S, K, T, r, sigma)  
print(f"Delta: {delta}")  
  
# Delta hedging: number of shares to hold  
num\_option\_contracts = 10  
hedge\_ratio = -delta \* num\_option\_contracts  
print(f"Hedge Ratio (shares to hold): {hedge\_ratio}")